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Some Modifications of Nano Bitopological Spaces

A. Pandi
Ettunali Pudur, Vidatha kulam(Post),
Thirumangalam(TK)- 625 706, Madurai District, Tamil Nadu, India.
e-mail : pandi2085@yahoo.com.

Abstract-In this paper, new classes of nano bitopological spaces and a new closed sets in nano bitopological spaces are introduced and its studied.

Key words- nano $(1,2)^*$-ψ-closed sets, nano $(1,2)^*$-$\hat{g}$-closed set, nano $(1,2)^*$-gs-closed set and nano $(1,2)^*$-αg-closed set.

1. INTRODUCTION

Levine [8] introduced the concept of generalized closed sets in topological spaces. Kelly [6] introduced the concepts of bitopological spaces. Ravi et al [11] and Ravi and Thivagar [10] introduced $(1,2)^*$ - α g-closed sets, $(1,2)^*$ -g-closed sets, $(1,2)^*$ -sg-closed sets and $(1,2)^*$ - $\hat{g}$ -closed sets respectively.

Lellis Thivagar et al [7] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space. Bhuvaneswari et.al [3] introduced and investigated nano g-closed sets in nano topological spaces.

In this paper, new classes of nano bitopological spaces and a new closed sets in nano bitopological spaces are introduced and its studied.

2. PRELIMINARIES

Definition 2.1 [9]
Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.

That is, $L_R(X) = \bigcup_{x\in U} \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U} \{R(x): R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [7] If (U, R) is an approximation space and $X, Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;

10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3 [7] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,

2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4 [7] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

3. MODIFICATIONS OF NANO BITOPOLOGICAL SPACES

Definition 3.1 Let U be a non-empty set, finite universe of objects and R_1, R_2 are two an equivalence relations on U. Let X_1, X_2 ⊆U. Let $\tau_{R_1}(X_1) = \{U, \phi, L_{R_1}(X_1), U_{R_1}(X_1), B_{R_1}(X_1)\}$. Then $(U, \tau_{R_1}(X_1))$ is a nano topological space and let $\tau_{R_2}(X_2) = \{U, \phi, L_{R_2}(X_2), U_{R_2}(X_2), B_{R_2}(X_2)\}$. Then $(U, \tau_{R_2}(X_2))$ is a another nano topological space. Then the triplet $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$ (briefly, $(U, \tau_{R_{1,2}}(X))$) is called the nano bitopological space.

Definition 3.2 Let U be a non-empty set, finite universe of objects and R_1, R_2 are two an equivalence relations on U. Let X_1, X_2 ⊆U. Let S be a subset of $(U, \tau_{R_1}(X_1), \tau_{R_2}(X_2))$. Then S is said to be $\tau_{R_{1,2}}(X)$-open if S = A ∪ B where A ∈ $\tau_{R_1}(X_1)$ and B ∈ $\tau_{R_2}(X_2)$.

The complement of $\tau_{R_{1,2}}(X)$-open set is called $\tau_{R_{1,2}}(X)$-closed.
Notice that $\tau_{R_{1,2}}(X)$-open sets need not necessarily form a nano topology.

Example 3.3 $U = \{a, b, c\}$ with $U/R_1 = \{\{a, b\}, \{c\}\}$ and $X_1 = \{a, b\}$. Then $\tau_{R_1}(X_1) = \{\phi, U, \{a, b\}\}$, $U/R_2 = \{\{a\}, \{b, c\}\}$ and $X_2 = \{b, c\}$. Then $\tau_{R_2}(X_2) = \{\phi, U, \{b, c\}\}$. Then $\tau_{R_{1,2}}(X) = \{\phi, U, \{a, b\}, \{b, c\}\}$ are $\tau_{R_{1,2}}(X)$-open sets but not form a nano topology, because {a, b} ∩ {b, c} = {b} not in $\tau_R(X)$.

Definition 3.4 [5] Let A be a subset of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$. Then

1. the nano $\tau_{R_{1,2}}$-closure of A, denoted by $N\tau_{R_{1,2}}$-cl(A), is defined as ∩ {F : A ⊆ F and F is $\tau_{R_{1,2}}(X)$-closed}.

2. the nano $\tau_{R_{1,2}}$-interior of A, denoted by $N\tau_{R_{1,2}}$-int(A), is defined as ∪ {F : F ⊆ A and F is $\tau_{R_{1,2}}(X)$-open}.

Definition 3.5 A subset A of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called

1. nano $(1,2)^*$-semi-open [4] if A ⊆ $N\tau_{R_{1,2}}$-cl($N\tau_{R_{1,2}}$-int(A));

2. nano $(1,2)^*$-α-open [2] if A ⊆ $N\tau_{R_{1,2}}$-int($N\tau_{R_{1,2}}$-cl($N\tau_{R_{1,2}}$-int(A)));

The complements of the above mentioned open sets are called their respective closed sets.

The nano $(1,2)^*$-semi-closure (resp. nano $(1,2)^*$-α-closure) of a subset S of U, denoted by N$(1,2)^*$-scl(A) (resp. N$(1,2)^*$-αcl(A)), is defined to be the intersection of all nano $(1,2)^*$-semi-closed (resp. nano $(1,2)^*$-α-closed) sets of $(U, \tau_{R_{1,2}}(X))$ containing A.

It is known that N $(1,2)^*$-scl(A) (resp. N$(1,2)^*$-αcl(A)) is a nano $(1,2)^*$-semi-closed (resp. nano $(1,2)^*$-α-closed) set.

Definition 3.6 A subset A of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called
1. nano $(1,2)^*$-g-closed [1] if $N\tau_{R_{1,2}}$-cl(A) ⊆ V whenever A ⊆ V and V is $\tau_{R_{1,2}}$-open in U.
The complement of nano $(1,2)^*$-g-closed set is called nano $(1,2)^*$-g-open set;

2. nano $(1,2)^*$-sg-closed [4] if N$(1,2)^*$-scl(A) ⊆ V

whenever A $\subseteq$ V and V is nano $(1,2)^*$-semi-open in U.

The complement of nano $(1,2)^*$-sg-closed set is called nano $(1,2)^*$-sg-open set;

Result 3.7
1. Every $\tau_{R_{1,2}}$-closed set is nano $(1,2)^*$-semi-closed but not conversely [4].

2. Every $\tau_{R_{1,2}}$-closed set is nano $(1,2)^*$-α-closed but not conversely [2].

3. Every nano $(1,2)^*$-semi-closed set is nano $(1,2)^*$-sg-closed but not conversely [4].

4. A NEW TYPE OF CLOSED SETS IN NANO BITOPOLOGICAL SPACES

Definition 4.1
1. nano $(1,2)^*$-gs-closed if $\mathrm{N}(1,2)^*$-scl(A) $\subseteq$ V whenever A $\subseteq$ V and V is $\tau_{R_{1,2}}$-open in U.

The complement of nano $(1,2)^*$-gs-closed set is called nano $(1,2)^*$-gs-open set;

2. nano $(1,2)^*$-αg-closed if $\mathrm{N}(1,2)^*$-αcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is $\tau_{R_{1,2}}$-open in U.

The complement of nano $(1,2)^*$-αg-closed set is called nano $(1,2)^*$-αg-open set;

3. nano $(1,2)^*$-$\hat{g}$-closed (or) nano $(1,2)^*$-ω-closed set if $\mathrm{N}\tau_{R_{1,2}}$-cl(A) $\subseteq$ V whenever A $\subseteq$ V and V is nano $(1,2)^*$-semi-open in U.

The complement of nano $(1,2)^*$-$\hat{g}$-closed (resp. nano $(1,2)^*$-ω-closed) set is called nano $(1,2)^*$-$\hat{g}$-open (resp. nano $(1,2)^*$-ω-open) set;

4. nano $(1,2)^*$-ψ-closed if $\mathrm{N}(1,2)^*$-scl(A) $\subseteq$ V whenever A $\subseteq$ V and V is nano $(1,2)^*$-sg-open in U. The complement of nano $(1,2)^*$-ψ-closed set is called nano $(1,2)^*$-ψ-open set.

Proposition 4.2 Every nano $(1,2)^*$-semi-closed set is nano $(1,2)^*$-ψ-closed.
Proof. If A is a nano $(1,2)^*$-semi-closed subset of U, $\mathrm{N}(1,2)^*$-scl(A) = A $\subseteq$ G whenever A $\subseteq$ G and G is nano $(1,2)^*$ -semi-open, since every nano $(1,2)^*$-semi-open set is $(1,2)^*$-sg-open. Hence A is nano $(1,2)^*$-ψ-closed.

The converse of Proposition 4.2 need not be true as seen from the following example.

Example 4.3 $U = \{p, q, r, s\}$ with $U/R_1 = \{\{p, q\}, \{r\}, \{s\}\}$ and $X_1 = \{p\}$. Then $\tau_{R_1}(X_1) = \{\phi, U, \{p, q\}\}$, $U/R_2 = \{\{p, r\}, \{q\}, \{s\}\}$ and $X_2 = \{s\}$. Then $\tau_{R_2}(X_2) = \{\phi, U, \{s\}\}$. Then $\tau_{R_{1,2}}(X) = \{\phi, U, \{s\}, \{p, q\}, \{p, q, s\}\}$. Clearly, the set {p, q, s} is a nano $(1,2)^*$-ψ-closed but it is not a nano $(1,2)^*$-semi-closed.

Proposition 4.4 Every nano $(1,2)^*$-$\hat{g}$-closed set is nano $(1,2)^*$-g-closed.

Proof. If A is a nano $(1,2)^*$-$\hat{g}$-closed subset of U and G is $\tau_{R_{1,2}}$-open set containing A, every $\tau_{R_{1,2}}$-open set is nano $(1,2)^*$-semi-open then $\mathrm{N}\tau_{R_{1,2}}$-cl(A) $\subseteq$ G. Hence A is nano $(1,2)^*$-g-closed.

The converse of Proposition 4.4 need not be true as seen from the following example.

Example 4.5 $U = \{a, b, c\}$ with $U/R_1 = \{\{a\}, \{b, c\}\}$ and $X_1 = \{a\}$. Then $\tau_{R_1}(X_1) = \{\phi, U, \{a\}\}$, $U/R_2 = \{\{a, c\}, \{b\}\}$ and $X_2 = \{a, c\}$. Then $\tau_{R_2}(X_2) = \{\phi, U, \{a, c\}\}$. Then $\tau_{R_{1,2}}(X) = \{\phi, U, \{a\}, \{a, c\}\}$. Clearly, the set {a, b} is an nano $(1,2)^*$-g-closed but not a nano $(1,2)^*$-$\hat{g}$-closed.

Proposition 4.6 Every nano $(1,2)^*$-sg-closed set is nano $(1,2)^*$-gs-closed.

Proof. If A is a nano $(1,2)^*$-sg-closed subset of U and G is any $\tau_{R_{1,2}}$-open set containing A, since every $\tau_{R_{1,2}}$-open set is nano $(1,2)^*$-semi-open, we have N $(1,2)^*$ -scl(A) $\subseteq$ G. Hence A is nano $(1,2)^*$-gs-closed.

The converse of Proposition 4.6 need not be true as seen from the following example.

Example 4.7 $U = \{a, b, c\}$ with $U/R_1 = \{\{a\}, \{b, c\}\}$ and $X_1 = \{a\}$. Then $\tau_{R_1}(X_1) = \{\phi, U, \{a\}\}$, $U/R_2 = \{\{a\}, \{b\}, \{c\}\}$ and $X_2 = \{a\}$. Then $\tau_{R_2}(X_2) = \{\phi, U, \{a\}\}$. Then $\tau_{R_{1,2}}(X) = \{\phi, U, \{a\}\}$. Clearly, the set {a, b} is an nano $(1,2)^*$-gs-closed but not a nano $(1,2)^*$-sg-closed.

Proposition 4.8 Every nano $(1,2)^*$-g-closed set is nano $(1,2)^*$-αg-closed.

Proof. If A is a nano $(1,2)^*$-g-closed subset of U and G is any any $\tau_{R_{1,2}}$-open set containing A, since every $\tau_{R_{1,2}}$-open set is nano $(1,2)^*$-α-open, we have N $(1,2)^*$ - α cl(A) $\subseteq$ G. Hence A is nano $(1,2)^*$-αg-closed.

The converse of Proposition 4.8 need not be true as seen from the following example.

Example 4.9 $U = \{a, b, c\}$ with $U/R_1 = \{\{a\}, \{b, c\}\}$ and $X_1 = \{a\}$. Then $\tau_{R_1}(X_1) = \{\phi, U, \{a\}\}$, $U/R_2 = \{\{a, b\}, \{c\}\}$ and $X_2 = \{a, b\}$. Then $\tau_{R_2}(X_2) = \{\phi, U, \{a, b\}\}$. Then $\tau_{R_{1,2}}(X) = \{\phi, U, \{a\}, \{a, b\}\}$. Clearly, the set {b} is an nano $(1,2)^*$-αg-closed but not a nano $(1,2)^*$-g-closed.

Proposition 4.10 Every nano $(1,2)^*$-g-closed set is nano $(1,2)^*$-gs-closed.

Proof. If A is a nano $(1,2)^*$-g-closed subset of U and G is any $\tau_{R_{1,2}}$-open set containing A, since every $\tau_{R_{1,2}}$-open set is nano $(1,2)^*$-semi-open, we have N $(1,2)^*$ -scl(A) $\subseteq$ G. Hence A is nano $(1,2)^*$-gs-closed.

The converse of Proposition 4.10 need not be true as seen from the following example.

Example 4.11 In Example 4.3. Clearly, the set {p} is an nano $(1,2)^*$-gs-closed but not a nano $(1,2)^*$-g-closed.

Remark 4.12 From the above Propositions, Examples and Remark, we obtain the following diagram -I and diagram-II, where A→B represents A implies B but not conversely.

$\tau_{R_{1,2}}$- closed → nano $(1,2)^*$-semi-closed → nano$(1,2)^*$-sg-closed
↓ ↓ ↓
nano $(1,2)^*$-α-closed nano $(1,2)^*$-ψ-closed nano$(1,2)^*$-gs-closed

Diagram-I

nano $(1,2)^*$-gs-closed
↓
nano $(1,2)^*$-$\hat{g}$-closed→ nano $(1,2)^*$-g-closed → nano$(1,2)^*$-αg-closed

Diagram-II

5. CONCLUSION

In this paper, new classes of nano bitopological spaces and a new closed sets in nano bitopological spaces are introduced and its studied.

In future, I will discuss more applications of nano topological spaces concepts applied in nano bitopological spaces.

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